HEAT CONDUCTION OF 2D COMPOSITE MATERIALS
WITH SYMMETRIC INCLUSIONS: A MODEL
AND REDUCTION TO A VECTOR-MATRIX PROBLEM

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**Abstract.** We consider steady potential heat conduction of a cylindrical composite material with the special geometry. The matrix is modelling by the unit disc with different conductivity of six equal sectors. Inclusions (having different conductivity too) are symmetrically situated discs non-intersecting boundary of sectors. Mixed boundary conditions on parts of the boundary of matrix and matrix-inclusions leads to different model of composite materials. A new method to study the corresponding mathematical model is proposed. It is based on the reduction of the problem to the vector-matrix boundary value problem for analytic vectors. The method is connected with the approach by Zhorovina and Mityushev to the study of \( \mathbb{R} \)-linear boundary value on a fan-shaped domain.

**Introduction**

We consider a steady heat conduction of two-dimensional composite materials with symmetrically situated inclusions. This work belongs to the analytic direction of the study of mathematical models for composite materials. We can mention here the results by P. Adler \cite{1,2}, G. P. Cherepanov \cite{3}, L. Berlyand \cite{4,5}, J. A. Kolodziej \cite{6}, R. C. MacPhedran \cite{7}, G. W. Milton \cite{8}, V. V. Mityushev \cite{9}, Yu. V. Obnosov and R. Craster \cite{10}, E. V. Pesetskaya \cite{11,12} et al. Our main idea is in wide use of the method of functional equations as developed in the recent monograph by V. Mityushev & S. Rogosin \cite{13}.

The composite material under discussion is a long cylinder (matrix) with the unit disc as the base having different (but constant) conductivities of six equal sectors. In each sector there is a cylindrical inclusion. All of them are parallel to the axis of the matrix, nonintersecting boundary of sectors and have different (constant) conductivities too. We study heat conduction in the potential case (on the section of the composite orthogonal to its axis) with ideal contact conditions either inbetween sector, or on the boundary of inclusions. The temperature distribution is supposed to be given on the outer boundary of the matrix. These conditions are rewritten in terms of complex potentials \cite{14}. Boundary conditions on the lines separated sectors of matrix and boundary conditions on the boundaries of inclusions become \( \mathbb{R} \)-linear conjugation conditions, and the condition on the outer boundary of the unit disc becomes the condition of the Schwarz boundary value problem.

Due to geometrical symmetry of the composite as well as symmetry in the prescribed conductivities of its different parts we can consider the only solutions satisfying special symmetry relations. By introducing new unknown vector-function we reduce equivalently the \( \mathbb{R} \)-linear
conjugation problems on the segments between sectors to the homogeneous Riemann vector-matrix boundary value problem with constant matrix on each segment. By another changing of unknown vector-function we arrive at the vector-valued conditions of analytic continuation on segments. Incorporating these new unknown vector-functions into remaining boundary conditions we have received an equivalent to starting problem a vector-matrix \( \mathbb{R} \)-linear conjugation problem on the multiply connected circular. The later is solved by the method of functional equations in complex domains in the monograph [13].

The proposed method is connected with the method for the study \( \mathbb{R} \)-linear boundary value on a fan-shaped domain [15], applied for the simple symmetric composite materials in [13]. It can by used also for the study of conductivity of more general symmetric 2D composite materials.

Mathematical Model

Let us give the geometric description of the model of a steady heat conduction of 2D composite material with the unit disc \( U \) in the section orthogonal to parallel cylindrical inclusions. Suppose that \( U \) is divided into 6 parts by the segments

\[
\Gamma_k := \{ z \in \mathbb{C} : 0 < |z| < 1, \ \arg z = 2\pi(k-1)/6 \}, \ k = 1, \ldots, 6.
\]

Inclusions \( \mathbb{D}_k \) encircled by the curve \( \mathbb{T}_k := \{ z \in \mathbb{C} : |z - a_k| = r \}, \ \arg a_k = \pi(2k-1)/6, |a_k| = r_0, 2r < r_0 < 1 - r \), are situated inside \( U \). The conductivity of the corresponding materials are \( \lambda_1 \) (for \( \mathbb{D}_1, \mathbb{D}_4 \)), \( \lambda_3 \) (for \( \mathbb{D}_2, \mathbb{D}_5 \)), \( \lambda_5 \) (for \( \mathbb{D}_3, \mathbb{D}_6 \)), respectively. Assume that the remaining parts of sectors \( (k = 1, \ldots, 6) \)

\[
\Omega_k := \{ z \in \mathbb{C} \setminus \mathbb{D}_k : 0 < |z| < 1, |z - a_k| > r, 2\pi(k-1)/6 < \arg z < 2\pi k/6 \},
\]

are filled in by materials of different conductivity, namely with coefficient \( \lambda_2 \) (for \( \Omega_1 \) and \( \Omega_4 \)), \( \lambda_4 \) (for \( \Omega_2 \) and \( \Omega_5 \)), and \( \lambda_6 \) (for \( \Omega_3 \) and \( \Omega_6 \)). Let \( \Gamma_k, \ k = 1, 2, 3 \), be oriented in the direction from the origin, and \( \Gamma_k, \ k = 4, 5, 6 \), be oriented in the direction to the origin. Let also \( \mathbb{T}_k \) be clock wise oriented, and the unit circle \( \mathbb{T} := \{ z \in \mathbb{C} : |z| = 1 \} \) be counter-clock wise oriented.

The mathematical problem is the following (cf. [13]): given a real-valued Hölder-continuous function \( f \in C^{1,\mu}(\mathbb{T}) \) find a (piece-wise) harmonic function \( u \in H^1_A(B), B := U \bigcup \left( 6 \bigcup \mathbb{D}_k \bigcup \mathbb{U} \right) \), and harmonic functions \( u_k \in H^1_A(\mathbb{D}_k) \) satisfying the conditions of ideal contact on the lines \( \Gamma_k \) and \( \mathbb{T}_k \), as well as condition of temperature distribution on the outer part of boundary:

\[
\begin{cases}
  u^+(t) = u^-(t), & \lambda_2 \frac{\partial u^+}{\partial n}(t) = \lambda_6 \frac{\partial u^-}{\partial n}(t), & t \in \Gamma_1, \\
  u^+(t) = u^-(t), & \lambda_4 \frac{\partial u^+}{\partial n}(t) = \lambda_2 \frac{\partial u^-}{\partial n}(t), & t \in \Gamma_2, \\
  u^+(t) = u^-(t), & \lambda_6 \frac{\partial u^+}{\partial n}(t) = \lambda_4 \frac{\partial u^-}{\partial n}(t), & t \in \Gamma_3, \\
  u^+(t) = u^-(t), & \lambda_6 \frac{\partial u^+}{\partial n}(t) = \lambda_2 \frac{\partial u^-}{\partial n}(t), & t \in \Gamma_4, \\
  u^+(t) = u^-(t), & \lambda_4 \frac{\partial u^+}{\partial n}(t) = \lambda_6 \frac{\partial u^-}{\partial n}(t), & t \in \Gamma_5, \\
  u^+(t) = u^-(t), & \lambda_4 \frac{\partial u^+}{\partial n}(t) = \lambda_6 \frac{\partial u^-}{\partial n}(t), & t \in \Gamma_6,
\end{cases}
\]

\[
\begin{cases}
  u^+(t) = u_k^+(t), & \lambda_2 \frac{\partial u^+}{\partial n}(t) = \lambda_1 \frac{\partial u^+_k}{\partial n}(t), & t \in \mathbb{T}_k, \ k = 1, 4, \\
  u^+(t) = u_k^+(t), & \lambda_4 \frac{\partial u^+}{\partial n}(t) = \lambda_3 \frac{\partial u^+_k}{\partial n}(t), & t \in \mathbb{T}_k, \ k = 2, 5, \\
  u^+(t) = u_k^+(t), & \lambda_6 \frac{\partial u^+}{\partial n}(t) = \lambda_5 \frac{\partial u^+_k}{\partial n}(t), & t \in \mathbb{T}_k, \ k = 3, 6.
\end{cases}
\]
\[ u(t) = f(t), ~ t \in \mathbb{T} \setminus \left\{ \bigcup_{j=1}^{6} e^{i(2\pi(j-1)/6)} \right\}. \] (3)

Boundary values are taken in accordance with the above fixed orientation, and \( h_{\lambda}^j \) are classes of functions which are harmonic in corresponding domains and continuously differentiable up to their boundaries.

Let us introduce complex potentials \( h(z), h_k(z) \) for each subdomain according to the following rule:

\[ u(z) := \text{Re} \ h(z), ~ z \in \bigcup_{k=1}^{6} \Omega_k, \] (4)

\[ u_k(z) := \text{Re} \ h_k(z), ~ z \in \mathbb{D}_k. \] (5)

In terms of complex potentials the boundary conditions (1)-(3) can be rewritten in the following form:

\[
\begin{cases}
    h^+(t) = \frac{\lambda_2 + \lambda_3}{2\lambda_2} h^-(t) + \frac{\lambda_2 - \lambda_3}{2\lambda_2} h^{-}(t), & t \in \Gamma_1, \\
    h^+(t) = \frac{\lambda_1 + \lambda_2}{2\lambda_1} h^-(t) + \frac{\lambda_1 - \lambda_2}{2\lambda_1} h^{-}(t), & t \in \Gamma_2, \\
    h^+(t) = \frac{\lambda_6 + \lambda_4}{2\lambda_6} h^-(t) + \frac{\lambda_6 - \lambda_4}{2\lambda_6} h^{-}(t), & t \in \Gamma_3, \\
    h^+(t) = \frac{\lambda_7 + \lambda_5}{2\lambda_7} h^-(t) + \frac{\lambda_7 - \lambda_5}{2\lambda_7} h^{-}(t), & t \in \Gamma_4, \\
    h^+(t) = \frac{\lambda_8 + \lambda_6}{2\lambda_8} h^-(t) + \frac{\lambda_8 - \lambda_6}{2\lambda_8} h^{-}(t), & t \in \Gamma_5, \\
    h^+(t) = \frac{\lambda_9 + \lambda_7}{2\lambda_9} h^-(t) + \frac{\lambda_9 - \lambda_7}{2\lambda_9} h^{-}(t), & t \in \Gamma_6, \\
    h^+(t) = \frac{\lambda_5 + \lambda_6}{2\lambda_5} h^-(t) + \frac{\lambda_5 - \lambda_6}{2\lambda_5} h^{-}(t), & t \in \mathbb{T}_k, ~ k = 1, 4, \\
    h^+(t) = \frac{\lambda_4 + \lambda_5}{2\lambda_4} h^-(t) + \frac{\lambda_4 - \lambda_5}{2\lambda_4} h^{-}(t), & t \in \mathbb{T}_k, ~ k = 2, 5, \\
    h^+(t) = \frac{\lambda_3 + \lambda_4}{2\lambda_3} h^-(t) + \frac{\lambda_3 - \lambda_4}{2\lambda_3} h^{-}(t), & t \in \mathbb{T}_k, ~ k = 3, 6,
\end{cases}
\] (6)

\[
\text{Re} \ h(t) = f(t), ~ t \in \mathbb{T} \setminus \left\{ \bigcup_{j=1}^{6} e^{i(2\pi(j-1)/6)} \right\}. \] (8)

**Reduction to a vector-matrix Riemann boundary value problem**

Now we reduce the problem (6)-(8) to a vector-matrix \( \mathbb{R} \)-linear problem for the domain \( \mathbb{D} := \mathbb{U} \setminus \left( \bigcup_{k=1}^{6} \mathbb{D}_k \right) \). First we introduce the following vector-function

\[
\Phi(z) := (\Phi_1(z), \ldots, \Phi_6(z))^T =
\]

\[
= (h(z), h(\overline{z}), h(ze^{i4\pi/3}), h(ze^{-i4\pi/3}), h(ze^{i2\pi/3}), h(ze^{-i2\pi/3}))^T.
\] (9)

Components of the vector \( \Phi \) satisfy the symmetry conditions

\[
\Phi_1(z) \equiv \Phi_2(\overline{z}) \equiv \Phi_3(ze^{i2\pi/3}) \equiv \Phi_4(ze^{-i4\pi/3}) \equiv \Phi_5(ze^{i4\pi/3}) \equiv \Phi_6(ze^{-i2\pi/3}). \] (10)

Identifying the corresponding points on the segments \( \Gamma_k, k = 1, \ldots, 6 \), and taking the complex conjugation in all the relation (6) we arrive at the following vector-matrix boundary value problem at each segment \( \Gamma_k, k = 1, \ldots, 6 \):
\[ \Phi^+(t) = G_k(t) \Phi^-(t), \quad t \in \Gamma_k, \quad k = 1, \ldots, 6, \quad (11) \]

\[ G_1 = \begin{pmatrix}
\frac{2\lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_4} & 0 & 0 & 0 & 0 \\
\lambda_4 + \lambda_6 & \lambda_2 + \lambda_4 & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_4} & 0 & 0 & 0 \\
\lambda_2 + \lambda_6 & \lambda_2 + \lambda_4 & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_4} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{2\lambda_2}{\lambda_2 + \lambda_4} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 \\
0 & 0 & 0 & \frac{\lambda_2 + \lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 \\
0 & 0 & 0 & \frac{\lambda_2 + \lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 \\
\end{pmatrix} \quad (12) \]

\[ G_2 = \begin{pmatrix}
\frac{2\lambda_4}{\lambda_2 + \lambda_4} & 0 & 0 & 0 & 0 & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_4} \\
0 & \frac{2\lambda_4}{\lambda_2 + \lambda_4} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_4} & 0 & 0 & 0 \\
0 & \frac{\lambda_2 + \lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 & 0 & 0 \\
0 & 0 & \frac{2\lambda_2}{\lambda_2 + \lambda_4} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 & 0 \\
0 & 0 & 0 & \frac{\lambda_2 + \lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 \\
0 & 0 & 0 & \frac{\lambda_2 + \lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 \\
\end{pmatrix} \quad (13) \]

\[ G_3 = \begin{pmatrix}
\frac{2\lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_4} & 0 & 0 & 0 & 0 \\
0 & \frac{2\lambda_4}{\lambda_2 + \lambda_4} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_4} & 0 & 0 & 0 \\
0 & \frac{\lambda_2 + \lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 & 0 & 0 \\
0 & 0 & \frac{2\lambda_2}{\lambda_2 + \lambda_4} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 & 0 \\
0 & 0 & 0 & \frac{\lambda_2 + \lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 \\
0 & 0 & 0 & \frac{\lambda_2 + \lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 \\
\end{pmatrix} \quad (14) \]

\[ G_4 = \begin{pmatrix}
\frac{2\lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_4} & 0 & 0 & 0 & 0 \\
0 & \frac{2\lambda_4}{\lambda_2 + \lambda_4} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_4} & 0 & 0 & 0 \\
0 & \frac{\lambda_2 + \lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 & 0 & 0 \\
0 & 0 & \frac{2\lambda_2}{\lambda_2 + \lambda_4} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 & 0 \\
0 & 0 & 0 & \frac{\lambda_2 + \lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 \\
0 & 0 & 0 & \frac{\lambda_2 + \lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 \\
\end{pmatrix} \quad (15) \]

\[ G_5 = \begin{pmatrix}
\frac{2\lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_4} & 0 & 0 & 0 & 0 \\
0 & \frac{2\lambda_4}{\lambda_2 + \lambda_4} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_4} & 0 & 0 & 0 \\
0 & \frac{\lambda_2 + \lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 & 0 & 0 \\
0 & 0 & \frac{2\lambda_2}{\lambda_2 + \lambda_4} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 & 0 \\
0 & 0 & 0 & \frac{\lambda_2 + \lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 \\
0 & 0 & 0 & \frac{\lambda_2 + \lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 \\
\end{pmatrix} \quad (16) \]

\[ G_6 = \begin{pmatrix}
\frac{2\lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_4} & 0 & 0 & 0 & 0 \\
0 & \frac{2\lambda_4}{\lambda_2 + \lambda_4} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_4} & 0 & 0 & 0 \\
0 & \frac{\lambda_2 + \lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 & 0 & 0 \\
0 & 0 & \frac{2\lambda_2}{\lambda_2 + \lambda_4} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 & 0 \\
0 & 0 & 0 & \frac{\lambda_2 + \lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 \\
0 & 0 & 0 & \frac{\lambda_2 + \lambda_4}{\lambda_2 + \lambda_6} & \frac{\lambda_2 - \lambda_6}{\lambda_2 + \lambda_6} & 0 \\
\end{pmatrix} \quad (17) \]
Introduce now new unknown vector-function
\[ F(z) = \begin{cases} 
\Phi(z), & z \in \Omega_1 \cup \mathbb{D}_1, \\
G_2^{-1}\Phi(z), & z \in \Omega_2 \cup \mathbb{D}_2, \\
G_2^{-1}G_3^{-1}\Phi(z), & z \in \Omega_3 \cup \mathbb{D}_3, \\
G_2^{-1}G_3^{-1}G_4\Phi(z), & z \in \Omega_4 \cup \mathbb{D}_4, \\
G_2^{-1}G_3^{-1}G_4G_5\Phi(z), & z \in \Omega_5 \cup \mathbb{D}_5, \\
G_2^{-1}G_3^{-1}G_4G_5G_6\Phi(z), & z \in \Omega_6 \cup \mathbb{D}_6.
\end{cases} \tag{18} \]

It gives that on each $\Gamma_k, k = 2, \ldots, 6$, the following relation take place:
\[ F^+(t) = F^-(t), \quad t \in \Gamma_k, k = 2, \ldots, 6, \]
and on $\Gamma_1$ we have
\[ F^+(t) = G_1G_2^{-1}G_3^{-1}G_4G_5G_6F^-(t), \quad t \in \Gamma_1. \tag{19} \]

It should be noted, that in the domain $\Omega_1$ the components of the vector $\Phi(z)$ satisfy the relations
\[ \Phi_1(z) \equiv \Phi_4(z), \quad \Phi_2(z) \equiv \Phi_5(z), \quad \Phi_3(z) \equiv \Phi_6(z), \tag{20} \]
but in $\Omega_6$ these relations have the form
\[ \Phi_1(z) \equiv \Phi_6(z), \quad \Phi_2(z) \equiv \Phi_3(z), \quad \Phi_4(z) \equiv \Phi_5(z). \tag{21} \]

Thus, the matrix $G_1G_2^{-1}G_3^{-1}G_4G_5G_6$ acts in these domains “as the unit matrix” since it satisfies the identity
\[ \begin{pmatrix} a \\ b \\ c \\ \overline{a} \\ \overline{b} \\ \overline{c} \end{pmatrix} = G_1 \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \overline{\alpha} \\ \overline{\beta} \\ \overline{\gamma} \end{pmatrix}. \]

It means that if the vector-function $\Phi$ satisfies the relations (20), (21), then the boundary condition (19) for components of the vector-function $F$ are in fact conditions of analytic continuation through $\Gamma_1$.

Therefore the problem (6)–(8) is equivalently reduced to the vector-matrix $\mathbb{R}$-linear conjugation problem on the multiply connected circular domain $\mathbb{U} \setminus \left( \bigcup_{k=1}^6 \mathbb{D}_k \right)$:
\[ \Phi^+(t) = H_k(t)\Phi^-(t) + S_k(t)\overline{\Phi}^-(t), \quad t \in \mathbb{T}_k, \quad k = 1, \ldots, 6, \tag{22} \]
\[ \Re \Phi(t) = \mathbf{q}(t), \quad t \in \mathbb{T} \setminus \left\{ e^{i(2\pi(j-1)/6)} \right\}, \tag{23} \]
where the constant matrices $H_k, S_k, k = 1, \ldots, 6$, are explicitly calculated via the matrices $G_k$, and vector-function $\mathbf{q}$ is defined by the formula
\[ \mathbf{q}(t) \equiv \left( f(t), \overline{f(t)}, f(\overline{t}e^{i2\pi/3}), f(\overline{t}e^{i4\pi/3}), f(\overline{t}e^{i6\pi/3}), \overline{f(\overline{t}e^{i2\pi/3})}, \overline{f(\overline{t}e^{i4\pi/3})}, \overline{f(\overline{t}e^{i6\pi/3})} \right)^T. \tag{24} \]

The problem (22), (23), (24) is solving in an appropriate functional spaces by a matrix variant of the functional equations method for complex domains as developed in the monograph [13].
Discussion and Outlook

The problem of heat conduction in a symmetric composite materials is considered. The method of the equivalent reduction of this problem to a vector-matrix $\mathbb{R}$-linear conjugation problem on a multiply connected circular domain is proposed. Its solution based on a vector variant of functional equations method is found. By using this solution the effective conductivity of such symmetric material can be calculated.

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References