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PORTFOLIO DISCRETE OPTIMIZATION PROBLEMS

Discrete optimization models play an increasingly role in financial decisions [1-4]. This report analyzes the portfolio discrete optimization models which is the most important of them. Portfolio optimization problems are based on mean-variance models for returns (linear models of returns, models with transactions costs [5], model using fuzzy expected return [6]) and for risk-neutral density estimation (model based on Euclidean metric of risk, model with risk-free asset — Tobin model, model based on Minkowski absolute metric of risk, model based on Minkowski semi-absolute metric of risk, model based on Chebyshev metric of risk — maxmin and minimax model).

The mathematical portfolio discrete optimization problems are the quadratic or linear parametrical programming with integer variables (model with integer assets — lots [7-8], model with limited number assets — cardinality constrained [9], model with buy-in thresholds). The mathematical problems can be formulated in many ways but the principal problems can be summarized as follows:

1. Bicriterial convex quadratic integer optimization with simple budget constraints;
2. Bicriterial linear integer optimization;
3. Linear integer optimization with simple polymatroidal budget and risk diversification constraints

We are discussing the mathematical models and modern optimization techniques for some classes of portfolio discrete optimization problems more important criteria: bicriterial (convex quadratic and linear — model Markowitz or model with risk-free asset — Tobin model), single (fractional — Sharpe model).

Simple greedy methods were offered for obtained fuzzy linear discrete optimization problem with polymartoidal constraints. The results were applied in banks for optimization of assets structure in foreign currencies and precious metals and also for optimization of bank asset and liability structure.

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